The Author Who Never Was: Nicolas Bourbaki

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"Secret societies." These words may call to mind two recent bestsellers, *The DaVinci Code*[1] and *Angels and Demons*.[2] These books tell the stories of ancient societies, shrouded in mystery, that used mathematics to protect themselves and their beliefs. Look into the more recent past, and you'll find another secret society, but a very different one.

Begun in France in the middle 1930s, this secret society has consisted of a small group of mathematicians—in the middle 1950s membership was estimated to be "about 12"[3]—who wrote a mathematical treatise "without which 20th-century mathematics would be . . . quite different from what it is".[4] The typical member was a graduate of the École Normale Supérieure, generally considered the most prestigious institution of tertiary education in France.[5] At its most productive, the group wrote one or two volumes per year.[3] Members have included some of the century's greatest mathematicians,[6] such as 1994 Kyoto Prize winner André Weil—also a group founder—yet no official membership list has ever been published.[7] And because new members have joined as old ones retired,[3] the group still exists. They call themselves Nicolas Bourbaki.

With authorship issues in scientific publication having received much recent attention,[8] the Bourbaki model of authorship seems especially interesting to consider. What was this group of mathematicians known as Bourbaki, and how did it begin? What did it accomplish and how? How did this unusual authorship structure affect interactions with publishers? And who had ownership rights to the books, and who

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received the royalties? In search of answers to those questions, I've read the accounts of Bourbaki members and interviewed former members and publishing professionals who have worked with Bourbaki.

The Founders and the Foundations

In late 1934, in Paris, a group of young mathematics professors who were educated at the École Normale Supérieure met in a café to discuss the writing of a mathematics treatise. That they would consider writing a text for a course one of them was teaching was not unusual: French university mathematics professors of the time did that regularly.[9] But the goal they would shortly decide to pursue—and estimate that they would complete "in about three years"[10]—was definitely unusual.

Their goal was the writing of a mathematical treatise, called *Éléments de Mathématique* (*Elements of Mathematics*), that would "provide a solid foundation for the whole body of modern mathematics".[11] The use of the singular *mathématique* reflected Bourbaki's belief in the unity of all of mathematics[12]: they sought to "expose the fundamental concepts common to all branches of mathematics".[13] The not-so-subtle play on the title of Greek mathematician Euclid's classic treatise on geometry, *Elements of Geometry*,[7] implied not only that their conception of mathematics was "illuminating and useful for dealing with the *current* concerns of mathematics, but that this was in fact the ultimate stage in the evolution of mathematics, bound to remain unchanged by any further development of this science".[14] The result was not intended to be a textbook "for everybody", but a reference, an "encyclopedia" of mathematics,[3] a "tool" for experts.[10] Bourbaki would start from scratch,[12] assuming very little, and proceed step-by-step to prove the rest. References were made only to other (earlier) works in this treatise—readers were to read

the volumes in a "rigorously fixed logical order"[4]—with the exception of the sections that provided historical context.[12] Bourbaki created its own terminology and notation to avoid the existing, often inexact use of language.[4] Because Bourbaki was "try[ing] to make each part of mathematics as general as possible in order to obtain the widest possible domain of applicability",[3] any heuristic remarks, to assist the reader, were "almost invariably thrown out" of proposed drafts as being "too vague, ambiguous, impossible to make precise in a few words". The resulting volumes were considered very abstract.[12] However, they also contained "Supplements"—sections that included "excellent" exercises for the reader.[4]

Given the circumstances, Bourbaki's goal made perfect sense. French mathematics suffered after World War I: Much of an entire generation of mathematicians who would have been doing research, as well as teaching and mentoring the generation to which Bourbaki belonged, had been killed during the war. Consequently, one founding member of Bourbaki noted that his generation had graduated from the École Normale Supérieure without being taught some basic mathematical concepts.[10] And generally, the existing study of mathematics in France lacked the "rigor" found in other countries, such as Germany, which was "dominating science at that time": "partly due to rancor after [World War I], people in the scientific establishment were not ready to accept the German method of science".[7] This group of young men, who would later be referred to as the "Founders",[15] would change that.

Because Bourbaki began as a "rebellion of a group of young people against the establishment", members wanted to protect their reputations and avoid attaching a long list of authors' names to their works. Therefore, they decided to give themselves a pseudonym. And anonymity was a small price to pay for the spirit of being "knights,

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heroes, . . . the champs, . . . the best, . . . because . . . we knew we were the authors of this famous series of books", commented Pierre Cartier, Bourbaki member between 1955 and 1983. "We took pride to achieve something almost perfect. . . . That was a strong reward." And perhaps belonging to a "secret society" was an appealing "romantic" notion, he added, although—while it was a subject of speculation—membership was never a complete secret.[7] Members chose the name Nicolas Bourbaki possibly on the basis of the statue of a General Bourbaki in the town of Nancy, where several Bourbaki members had taught; "Nicolas" may allude to the bringing of gifts by St Nicolas.[16]

The actual pace at which these "gifts" were distributed was much slower than the members initially estimated.[10] For example, it took 4 years before the first volume, consisting of a single chapter, was published in 1939.[15] (The chapters, intended to be self-contained units, are typically 200-250 pages long,[7] and "books" often consist of more than one published volume. For example, Bourbaki's Book II, on algebra, contains 10 chapters and was published as more than one volume.) The delays have been related to Bourbaki's method of working. In the spirit of their collaborative effort, the founders decided that any member could be asked to work on a draft, known as a "redaction"; one retired member reports that "the first [draft] was written by a specialist, but anyone might be asked to write a later one".[12] Typically the group met three times per year—once for 2 weeks and twice for 1 week—and writings were read aloud to all members. In the same collaborative spirit, everyone was expected to voice his opinions freely during the meetings—also known as "congresses"—which led to a generally chaotic atmosphere with members often yelling their opinions at the same time. That led to rewrites, or outright rejections, of works. A nonmember summarized the scene of the first meeting he attended: "Two or three monologues shouted at top voice, seemingly independently of one another."[12]

A nonmember? Yes, wives (all members

have been male[7]) and a few other outsiders were invited to attend as guests and advisers.[9] In addition, the Founders decided that members of Bourbaki must retire by 50, reflecting their desire to maintain a membership doing the most current work in the field.[10] Therefore, a potential new member—referred to as a *cobaye* or guinea pig—was invited to attend the Bourbaki meetings. If he participated in the discussions, he might be invited back and eventually be made a member.[10] A few mathematicians would join the initial generation.[7] In the years after World War II, a second generation, referred to as the "Middles"[15]—generally promising students of the Founders—would be selected in that way.[7]

Given the chaos of the meetings—and the requirement that all decisions be unanimous[12]—it may seem surprising that the group could agree on a single version, let alone publish anything. It is true that each chapter went through numerous revisions, sometimes ending up closer to the original draft than the intermediate drafts. But members would say that the more contentious a discussion, the more fruitful it was: "The underlying thought", commented Armand Borel, Bourbaki member from 1949 to 1973, "was apparently that really new, groundbreaking ideas were more likely to arise from confrontation than from an orderly discussion." (Members reportedly remarked that "l'esprit a soufflé"—"the spirit has blown"—after particularly heated discussions.[12])

Other, perhaps more mundane tasks have also been required to turn the drafts into published works. A nonmember has almost always been employed as a secretary, whose work has included typing, keeping the members informed through correspondence, and arranging the Bourbaki "congresses". Bourbaki members have performed such tasks as managing the group, noting changes in drafts during the meetings, writing or typing up the final manuscript, dealing with the publisher, and proofreading the publisher's final galleys and page proofs. Finally, a few members were considered over the years to be the informal "scientific leaders" of the

group, who informally guided the group's mathematical focus.[7] (André Weil was the first such leader; third-generation "Young One"[15] Pierre Cartier was another.)

Although most members took on at most one or two of those tasks for part or all of their membership, a few members were especially active. They included Founder Jean Dieudonné, who retired in the 1950s. In addition to his work drafting some chapters,[7] he "took care of the final drafts, exercises, and preparation for the printer of all of the volumes (about thirty) which appeared while he was a member and even slightly beyond".[12] He also read proofs from the publisher.[7] By virtue of that work, a uniform style—which Dieudonné "adopted for Bourbaki"—was imposed on the books during his tenure.[12]

Because of the obvious importance to Bourbaki of every word in their books, they needed to work with a publisher that would allow them to maintain their control. After publishing a few papers in mathematics journals,[4] Bourbaki selected the highly respected scientific publisher Hermann, who was given the copyrights to Bourbaki's treatise.[5]

Founder Jean Delsarte was the first long-term "manager" of Bourbaki, in Cartier's words. In that capacity, Delsarte signed the first contract with Hermann and therefore received the royalties.[7]

A "Golden Age": The 1950s and 1960s

Although work began on the *Éléments* after the middle 1930s, most of Bourbaki's work would not start to appear until after World War II, and not appear in earnest until the 1950s and 1960s. Why? For one thing, the process of writing each chapter took a great deal of time, as has already been noted. In addition, even though members continued to work during the war, and Delsarte managed to maintain communication with most Bourbaki members—including, for a time, those who had been forced to escape from France or go into hiding—no volumes could be printed.[7]

After the war, "enough material was completed that six or seven volumes could be published over the next two years", Cartier noted, and Bourbaki could again focus on the task of writing and publishing the *Éléments*. This was so huge a task that even with the addition of new Bourbaki members after the war, members could easily choose to work on the subjects that interested them most; in Cartier's words, everyone had "enough to eat".[7]

And eat they did! By the end of the 1950s, due to Bourbaki's efforts, including those of especially prolific members like Dieudonné, work on the fundamental material—essentially material that a master's degree mathematics student would be expected to know[3]—would be "essentially finished".[12] So much work was done that, through the 1950s and 1960s—a period referred to by Cartier as Bourbaki's "golden age"[7]— Hermann would publish one or two new volumes per year, totaling more than 30 volumes that comprised Bourbaki's six fundamental Books, including Book I, *Théorie des Ensembles* (*Set Theory*), and Book II, *Algèbre* (*Algebra*). At the same time, "all the books went through many editions; they were revised, enhanced, and improved".[7]

As the volumes were published, they became very popular—selling out and requiring reprints to be published—and Bourbaki became very influential.[7] Mathematician Ralph Boas Jr noted some of the reasons: Bourbaki "gave the first systematic account of some topics that previously had been available only in scattered articles", and the approach was also particularly appealing to pure mathematicians. Also, "since mathematicians had to learn Bourbaki's terminology in order to read his work, that terminology has become widely known and has changed much of the vocabulary of research".[16] Cartier, who joined Bourbaki in 1955, noted that even the initial volumes published after the war "had an enormous impact on my generation. That's where I really learned mathematics. . . . [Those volumes] helped very much to establish the fame of Bourbaki and the power of Bourbaki. . . . My generation was attracted to [the academic power of Bourbaki]. And on the other hand [Bourbaki members] were very prominent mathematicians who achieved very high positions in the French academic system."[7]

Even in the early 1950s, the royalties from the sale of the *Éléments* were large enough to increase Delsarte's taxes. Something needed to be done. Cartier half-jokingly stated that "if [members] had to divide the royalties, it would start fights. . . . The best way to protect the peace of the group" was to become a nonprofit organization, requiring Bourbaki to show that all profits were used for the group's expenses, including the secretary's salary, typewriters (and eventually computers), and all the expenses for members to attend the three yearly meetings. In that way, the Association des Collaborateurs de Nicolas Bourbaki (Association of the Collaborators of Nicolas Bourbaki), became an official entity, requiring, at least in name, a president, treasurer, and secretary.[7]

In the middle 1960s, Hermann sold the English rights to American publisher Addison-Wesley and arranged for them to publish translations of the books.[5] (Translations have also appeared in other languages, including Russian, German, and Spanish.)

Bourbaki would also work with a few other publishers in connection with another activity they were undertaking: Beginning in 1948, Bourbaki began the distinguished triannual Séminaire Nicolas Bourbaki (the Bourbaki seminar). "These are international conferences"[17] where mathematicians present either their own recent work or that of other mathematicians. Bourbaki selects the topics[17] and has the proceedings published. (In the 1950s, "about half of the papers were written by members of Bourbaki", some of whom "published in the seminar series part of their discoveries, or preliminary accounts of Bourbaki's ideas that later appeared in the books". Today, Cartier notes, "it's still a distinguished series but it's usually written by people who have no direct connection with . . . Bourbaki".[3]) The publishers of the seminars have included W A Benjamin, Inc, in New York; Springer-Verlag; and Société Mathématique de France.[18]

While Bourbaki grew in influence, the group "seem to have become more and more enamored of their joke [of using a pseudonym] and have often tried to

Bourbaki congress at Pelvoux, June-July 1951. Left to right: Jean Dieudonné, Jacques Dixmier, André Weil, Laurent Schwartz, and Roger Godement. Photo courtesy of Henri Cartan.

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persuade people that there is indeed an individual named N. Bourbaki, who writes books". One story that Ralph Boas Jr tells is about the time he referred to Bourbaki as a "collective pseudonym" in an *Encyclopaedia Britannica* article. Bourbaki sent letters of protest to both the encyclopedia and Boas; to the latter, Bourbaki demanded, "You miserable worm, how dare you say that I do not exist?" In response to pressure to settle the question of Bourbaki's existence, Boas referred *Encyclopaedia Britannica* Editor Walter Yust to American Mathematical Society Secretary J R Kline, who noted that an earlier application from Bourbaki for membership in the American Mathematical Society was rejected on the ground that he was not an individual. Later, Boas noted, "I was told that Bourbaki had tried to float a rumor that Boas was merely a collective pseudonym of the editors of *Mathematical Reviews*."[16]

A Change of Direction and Publishers

Amid Bourbaki's success and humorous antics, a critical issue was arising: Once the six fundamental books were written in the 1950s, what should Bourbaki write about next? Cartier commented that "the first generation had first to create a project from nothing. They had to invent a method. Then in the forties . . . the method had emerged and Bourbaki knew where to go: His goal was to provide a foundation for mathematics. . . . What should be included was more or less clear. . . . The third generation had to go beyond that. . . ."[3] Retired member Borel noted that mathematics had "grown enormously". And with the retirement of the first generation of Bourbaki, the next generation could question traditions, such as proceeding only linearly from one topic to the next. Inspired in part by Bourbaki's work, good materials now existed on topics that Bourbaki had not yet addressed; should Bourbaki write about these topics as well while postponing writing about some other important ones? "Two tendencies, two approaches, emerged: one . . . to go on building up broad foundations in an autonomous way, in the tradition of

Bourbaki; the other, more pragmatic, to get to [worthwhile] topics we felt we could handle, even if the foundations had not been thoroughly laid out in the optimal generality."[12]

After long discussion, the members reached a decision that allowed them to move forward—they would lay just enough groundwork to pursue subjects "in which they felt the Bourbaki approach might produce useful expositions". For 2 decades, beginning in the early 1960s, they would write another two books, plus two chapters of a third book.[12]

As Bourbaki began "dealing with more specialized topics closer to the front", it became more difficult to maintain the ideal that any member should be able to write on any topic: although "the rule was quite strict in the beginning . . . it became more lax."[12] Bourbaki also received the occasional assistance of nonmembers. A few nonmembers' contributions were considered important enough that their work is credited in the books, but this was not typical.[7] Bernard Teissier, a Bourbaki member from 1974 to 1995, noted that, more commonly, any assistance—including writing partial redactions or suggesting exercises—from nonmembers was accepted as "friendly help for a good cause, without acknowledgement. . . . Members in the process of writing a redaction would occasionally show it to some expert to get his impression."[19] In Cartier's words, "the border [between members and nonmembers] was never completely defined".[7]

In addition to deciding on the new topics to cover, Bourbaki decided to "revise the *Éléments* [once more] and publish a 'final' edition, not to be tampered with for at least fifteen years".[12] Consequently, some topics were "developed and deepened", some proofs were improved, some exercises were added, and a small volume was published in an attempt to "correct one obviously mistaken point of view of Bourbaki".[3] (Bourbaki is still revising parts of the *Éléments de Mathématique*.[20])

While that work was proceeding, the good relationship that Bourbaki had maintained for many years with the publisher

Hermann[21] eventually ended in the middle 1970s in disputes over royalties and translation rights.[3] After an extended legal battle between Bourbaki and Hermann, which ended in 1980, Bourbaki regained all rights to ownership of their works, as well as any unsold or unbound volumes.[7] At that point, Bourbaki began working with French publisher Masson, which published five or six volumes that Bourbaki had been worked on during the lawsuits and the volumes returned by Hermann.[3] Springer became the publisher of the English translations. Bourbaki would never again "give . . . any . . . rights [to the entire series] on the basis of . . . experience" with Hermann, preferring to conclude individual contracts for new books or for new editions of existing books "one-by-one" with the publisher.[5]

Cartier later observed that the time that Bourbaki devoted to determining their future directions and the lawsuit with Hermann "slowed the momentum of the group",[8] and he lists the evidence: Bourbaki's last new work—consisting of 187 pages—was published in 1998, only 200 copies were printed, it was not reviewed in any journals, and most of it was written in the early 1980s! Currently, there is no French publisher of Bourbaki's work; in the same year that the last volume was published, Masson decided not to publish any more of Bourbaki's works. Finally, only 10 of the 27 volumes published by Masson are currently in print.[7]

Bourbaki Reaches 70

Retired member Armand Borel stated in 1998 that "enough was carried out [by Bourbaki], it seems to me, to have a lasting impact on mathematics by fostering a global vision of mathematics and of its basic unity and also by the style of exposition and choice of notation. . . ."[12] One small measure of this is that some of Bourbaki's terminology and notation has become standard mathematical notation. (For example, as you may recall if you studied "sets" in a mathematics course, the symbol for the set that contains nothing is Ø; it was adopted by Bourbaki from the Norwegian alphabet.[22])

What about the present? Although some suggested as early as the middle 1980s—when Bourbaki was approaching his 50th year—that it was time for Bourbaki to retire,[7] he is still alive, even if not all believe he is well.

As recently as 2004, Springer published an English translation of a Bourbaki volume. But Springer editor Byrne has noted a drop in the sale of Bourbaki books over the last 15 years,[5] causing the book price to increase—as of early 2005, one recently published hardcover volume sold for €106.95.[23] In Byrne's opinion, the drop in sales is because Bourbaki's approach represents "an age that is past": Geometric methods and intuition have become more popular approaches in mathematics textbooks, and subjects that Bourbaki ignored—such as probability—have become very important in mathematics.[5]

Bourbaki continues to revise the *Éléments de Mathématique*, although the exercise is not as intense or as voluminous as in the distant past. At the moment, Bourbaki Secretary Viviane Le Dret reports that "a new completely revised version of *Algebra*, Chapter 8, is forthcoming, and N Bourbaki is earnestly rewriting Chapter 11 of *General Topology*". In addition, the Archives de la Création Mathématique, a French Centre National de la Recherche Scientifique research unit, has scanned some 200 redactions.[20]

In conclusion, for 70 years, Bourbaki has provided an intriguing model of authorship, one in which individuals have been willing to devote a remarkable amount of time and energy anonymously to a cause in which they believe, working with fellow devotees, expanding their own mathematical knowledge, and engaging in deep thought and loud arguments. (The prestige that would eventually come from belonging to Bourbaki was an added benefit.) Other groups of mathematicians have worked together anonymously, including H Petard[4] and John Rainwater,[24] but the magnitude of Bourbaki's work and impact on mathematics appears to be unique and may remain so. University of Michigan associate dean and mathematics professor Robert Megginson said that Bourbaki members' willingness to contribute anonymously to such an endeavor is not likely to occur in the current culture where individuals want and (because of competition for jobs) need to be acknowledged for their work.[25]

Has Bourbaki done what it set out to accomplish? In Cartier's words, "Its [initial] stated goal, to provide foundations for all existing mathematics, was achieved."[3] But, beyond that, Borel commented, "Bourbaki has not realized all its dreams or reached all of its goals by far." For example, he thinks that "we might have written more books . . . . [In fact,] there is a tremendous amount of unused material [written by Bourbaki, sitting] in Bourbaki's archives".[12] What's next for Bourbaki? Will the motivation continue or will a new motivation present itself? Regardless of the answers to those and other questions, the interest in Bourbaki's work continues: in February 2000, *Pour la Science*, the French version of *Scientific American*, devoted the second issue of its series *Les Génies de la Science* (*The Geniuses of Science*) to Bourbaki.[26] And who knows: This mathematician who never existed may yet celebrate "his" 100th birthday. 🔥

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